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Spherically symmetric electromagnetic mass models of embedding class one

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Abstract In this article we consider the static spherically symmetric spacetime metric of embedding class one. Specifically three new electromagnetic mass models are derived where the solutions are entirely dependent on the electromagnetic field, such that the physical parameters, like density, pressure etc. do vanish for the vanishing charge. We have analyzed schematically all these three sets of solutions related to electromagnetic mass models by plotting graphs and shown that they can pass through all the physical tests performed by us. To validate these special type of solutions related to electromagnetic mass models a comparison has been done with that of compact stars and shown exclusively the feasibility of the models.

Keywords General Relativity; equation of state; electromagnetic mass; compact stars

1 Introduction

It is a widely accepted concept that the n dimensional manifold V_n can be embedded in a pseudo-Euclidean space of $m = n(n+1)/2$ dimensions. The minimum extra dimensions, $m - n = n(n-1)/2$ of the pseudo-Euclidean space needed is called the embedding class of V_n. In case of the 4 dimensional relativistic spacetime, the embedding class is obviously 6. The well-known cosmological metric of FriedmannLemaîtreRobertsonWalker (FLRW) [1] is of class 1, whereas the Schwarzschild's interior and exterior solutions are of class 1 and 2 respectively. The Kerr spacetime metric has been shown to be of class 5 [2]. However, in the present paper we are limiting ourselves to the static spherically symmetric metric of embedding class 1 spacetime.

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It is seen that the above mentioned metric is compatible only with two perfect fluid distributions, viz. (i) Schwarzchild's solution [3] and (ii) Kohler and Chao [4] solution. We would like to exploit this metric to construct *electromagnetic mass* models under the Einstein-Maxwell framework by considering a charged perfect fluid distribution. In general, when the charge is zero in a charged distribution of matter, the subsequent distribution becomes the neutral counterpart of the charged distribution. This neutral counter part may belongs to the type of either Schwarzschild interior solution [3] or Kohler-Chao interior solution [4].

However, every charged fluid distribution indeed does not possess its neutral counter part and consequently if the charge is set to zero then the describing metric turns out to be flat and the corresponding energy density and fluid pressure will vanish identically. This special type of charged fluid distribution is said to provide an electromagnetic mass model. In connection with his model for extended electron Lorentz [5] conjectured that there is no other, no true or material mass, and thus proposed electromagnetic masses of the electron. Later on Wheeler [6] and Wilczek [7] pointed out that electron has a mass without mass. Feynman, Leighton and Sands [8] actually termed this type of models as electromagnetic mass models. For further reading on historical notes and technical works on the electromagnetic mass one may look at the Ref. [9] and [10, 11, 12, 13, 14, 15, 16, 17, 18, 19] respectively under the framework of Einstein-Maxwell theory.

Unfortunately, the electromagnetic mass models proposed by most of the above investigators [11, 12, 13, 14, 15, 16, 17, 18, 19] suffer from a negative pressure or density of the fluid due to the equation of state of the form

$$\rho + p = 0, \tag{1}$$

where ρ is the density and p is the pressure. This type of equation of state in the literature known as a false vacuum or degenerate vacuum or ρ-vacuum [20, 21, 22, 23]. It has been argued that though, in general, this equation of state leads to negative pressure but provides easier junction conditions and realistic expression for mass [12, 13, 14, 24]. Although the junction conditions do not require the density to vanish at the boundary as is true for gaseous spheres. Such a model is available in the literature for both uncharged and charged cases [25, 26]. However, we also note that the classical models of electron should contain the regions of negative density [27, 28]. It would be interesting to mention that a Weyl-type character of the field has been attributed which form electromagnetic mass model [29].

In the present study we have attempted to obtain a charged fluid of class 1 by choosing specific metric potential(s) of the class 1 such that they do not form a subset of the metric potentials of Schwarzschild's interior metric (inclusive de-Sitter and Einstein universe) and Kohler-Chao metric [4]. We argue here that the static spherically symmetric metric of embedding class one is more suitable to construct electromagnetic mass model as it possess lesser number of neutral counterparts of the charged fluids in comparison to general static spherical symmetric metric. Now if the charge be zero in the charged fluid, the describing metric will turn into flat by virtue of the structure of the metric. In the past, several alternatives were used by several investigators to obtain the electromagnetic mass models [12, 13, 14, 15, 16] by employing the equation of state (1) as pure charge condition [13] which takes the equivalent form as $g_{11}g_{44} = -1$ [12]. On the other hand, Ponce de Leon [16] has utilized the charged Einstein's clusters [30, 31] to get the electromagnetic mass

models. For further studies on different aspects of electromagnetic mass models one may look at the Refs. [32,33,34,35,36,37,38].

However, for the construction of electromagnetic mass models we invoke a different method by adopting an algorithm which is very efficient to generate solutions of the desired form and physics, as such no *ad hoc* assumptions are required to obtain electromagnetic mass models. The main motivation of the present paper, therefore, is to obtain a set of solutions for the electromagnetic mass model with the help of charged fluid distribution of spherically symmetric class one metric. The logic behind considering the class one metric is that if one removes charge from the solutions then either the Schwarzschild solution [3] or Kohler-Chao solution [4] will emerge the metric being flat and all the physical parameters - pressure, density etc. - become zero. To this aim our scheme of investigations are as follows: in Sect. 2 we provide the class one metric and fit the metric potentials in to the Einstein-Maxwell field equations for the spherically symmetric matter distribution. The next part is to construct electromagnetic mass models for stellar systems we provide the necessary algorithm (Sect. 3) and by exploiting the mathematical formalism we generate three new set of solutions in connection with electromagnetic mass models (Sect. 4). We also discuss the boundary conditions regarding all these solutions and determine the unknown constants of integration (Sect. 6). As till now we don't know exact nature of the solutions set so we adopt in Sect. 7 some specific techniques to explore the different features and properties of the electromagnetic mass models for physical acceptability of the anisotropic stellar models. In Sect. 8 we try to validate the solutions set related to electromagnetic mass models with some of the observed compact star candidates. We discuss our results in the concluding Sect. 9.

2 The class one metric and the Einstein-Maxwell field equations

Let us consider the static spherical symmetric metric to be

$$ds^2 = -e^{\lambda}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2) + e^{\nu}dt^2, \tag{2}$$

which may represent the space time of emending class 1, if it satisfies the Karmarker condition [39]

$$R_{1414} = \frac{R_{1212}R_{3434} + R_{1224}R_{1334}}{R_{2323}}, \tag{3}$$

with $R_{2323} \neq 0$ [40].

The above condition along with (2) yields the following differential equation

$$\frac{\lambda'\nu'}{(1-e^{\lambda})} = -2\left(\nu'' + \nu'^2\right) + \nu'^2 + \lambda'\nu', \tag{4}$$

with the constraint $e^{\lambda} \neq 1$, where λ and ν are metric potentials of the line element (2) which are function of radial coordinate r only.

The solution of the above differential equation (4) can be obtained as

$$e^{\lambda} = \left(1 + K\frac{\nu'^2 e^{\nu}}{4}\right), \tag{5}$$

where K is non zero arbitrary constant, $\nu'(r) \neq 0$, $e^{\lambda(0)} = 1$ and $\nu'(0) = 0$.

If Eqs. (2) and (5) describe a charge perfect fluid distribution then the functions $\lambda(r)$ and $\nu(r)$ must satisfy the Einstein-Maxwell field equations

$$G^i{}_j = R^i{}_j - \frac{1}{2}Rg^i{}_j = \kappa(T^i{}_j + E^i{}_j), \tag{6}$$

where $\kappa = 8\pi$ is the Einstein constant with $G = c = 1$ in the relativistic geometrized units.

The matter within the star is assumed to be locally a perfect fluid and consequently T^i_j and E^i_j, the energy-momentum tensors for the fluid distribution and the electromagnetic field tensors, are respectively defined by

$$T^i{}_j = [(c^2\rho + p)v^i v_j - p\delta^i{}_j, \tag{7}$$

$$E^i{}_j = \frac{1}{4\pi}(-F^{im}F_{jm} + \frac{1}{4}\delta^i{}_j F^{mn}F_{mn}), \tag{8}$$

where v^i is the four-velocity as $e^{\lambda(r)/2}v^i = \delta^i{}_4$, θ^i is the unit space-like vector in the direction of radial vector, $\theta^i = e^{\lambda(r)/2}\delta^i{}_1$, ρ is the matter-energy density and p is the fluid pressure.

The above anti-symmetric electromagnetic field tensor F_{ij} in Eq. (8), denotes the velocity and can be defined

$$F_{ij} = \frac{\partial A_j}{\partial A_i} - \frac{\partial A_i}{\partial A_j}, \tag{9}$$

This should satisfy the Maxwell equations

$$F_{ik,j} + F_{kj,i} + F_{ji,k} = 0, \tag{10}$$

and

$$\frac{\partial}{\partial x^k}(\sqrt{-g}F^{ik}) = -4\pi\sqrt{-g}J^i, \tag{11}$$

where g is the determinant of quantities g_{ij} in Eq. (11) and is given by

$$g = \begin{pmatrix} e^\nu & 0 & 0 & 0 \\ 0 & -e^\lambda & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 sin^2\theta \end{pmatrix} = -e^{(\nu+\lambda)}r^4 sin^2\theta,$$

where $A_j = (\phi(r), 0, 0, 0)$ is the four-potential and J^i is the four-current vector defined by

$$J^i = \frac{\sigma}{\sqrt{g_{00}}}\frac{dx^i}{dx^0} = \sigma v^i,$$

where σ is the charged density.

For static matter distribution the only non-zero component of the four-current is J^4 and because of the spherical symmetry this has only a functional relation with the radial coordinate r. The only non-vanishing component of the electromagnetic field tensor ($F^{41} = -F^{14}$) describes the radial component of the electric field. Hence, from Eq. (11), one can easily get the expression for the electric field

$$F^{41} = e^{-(\nu+\lambda)/2}\left[\frac{q(r)}{r^2}\right], \tag{12}$$

where $q(r)$ represents the electric charge contained within the sphere of radius r and is defined by

$$q(r) = 4\pi \int_0^r \sigma r^2 e^{\lambda/2} dr = r^2 \sqrt{-F_{14}F^{14}} = r^2 F^{41} e^{(\nu+\lambda)/2}. \tag{13}$$

Equation (13) can be treated as the relativistic version of Gauss's law which reduces to the following form:

$$\frac{\partial}{\partial r}(r^2 F^{41} e^{(\nu+\lambda)/2}) = -4\pi r^2 e^{(\nu+\lambda)/2} J^4. \tag{14}$$

For the spherically symmetric metric (2), the Einstein-Maxwell field equations can be expressed in the following ordinary differential equations

$$-\kappa T^1{}_1 = \frac{\nu'}{r} e^{-\lambda} - \frac{(1-e^{-\lambda})}{r^2} = \kappa p - \frac{q^2}{r^4}, \tag{15}$$

$$-\kappa T^2{}_2 = -\kappa T^3{}_3 = \left[\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu'-\lambda'}{2r}\right] e^{-\lambda} = \kappa p + \frac{q^2}{r^4}, \tag{16}$$

$$-\kappa T^4{}_4 = \frac{\lambda'}{r} e^{-\lambda} + \frac{(1-e^{-\lambda})}{r^2} = \kappa\rho + \frac{q^2}{r^4}, \tag{17}$$

where the prime denotes differentiation with respect to the radial coordinate r.

By using the Eqs. (15), (16), (17) and also (5), we obtain

$$\frac{\nu'}{r^2\left(4+K\nu'^2 e^{\nu}\right)}(4r - K\nu') = \kappa p - \frac{q^2}{r^4}, \tag{18}$$

$$\frac{4}{\left(4+K\nu'^2 e^{\nu}\right)}\left[\frac{\nu'}{2r} - \frac{\left(K\nu' e^{\nu} - 2r\right)\left(2\nu'' + \nu'^2\right)}{2r\left(4+K\nu'^2 e^{\nu}\right)}\right] = \kappa p + \frac{q^2}{r^4}, \tag{19}$$

$$\frac{K\nu' e^{\nu}}{\left(4+K\nu'^2 e^{\nu}\right)}\left[\frac{4(2\nu''+\nu'^2)}{\left(4+K\nu'^2 e^{\nu}\right)} + \frac{\nu'}{r}\right] = \kappa\rho + \frac{q^2}{r^4}. \tag{20}$$

On the other hand, the pressure isotropy condition can be given by

$$\left(\frac{k\nu' e^{\nu}}{2r} - 1\right)\left[\frac{2\nu'}{r\left(4+K\nu'^2 e^{\nu}\right)} - \frac{4(2\nu''+\nu'^2)}{\left(4+K\nu'^2 e^{\nu}\right)^2}\right] = \frac{2q^2}{r^4}. \tag{21}$$

A closer observation of the above set of differential equations easily indicates that if charge vanishes in a charged fluid of embedding class one, then survived neutral counterpart will only be either the Schwarzschild [3] interior solution (or its special cases de-sitter universe or Einstein's universe) or the Kohler-Chao [4] solution, otherwise either the charge cannot be zero or the survived space-time metric will become flat.

Now, one can look at Eq. (21) which immediately indicates that in absence of the charge either of the two factors on the left hand side has to be zero. Consequently, it can be shown that if the first factor of Eq. (21) be zero then it gives rise to the Kohler-Chao [4] solution in the form:

$$ds^2 = -\frac{(A+2Br^2)}{(A+Br^2)} dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2) + (A+Br^2)dt^2. \tag{22}$$

The pressure and density, in this model, are

$$\kappa p = \frac{B}{(A + 2Br^2)}, \tag{23}$$

$$\kappa \rho = B\frac{(3A + 2Br^2)}{(A + Br^2)}. \tag{24}$$

One can observe from (23) that since it does not possess zero pressure as well as density for any finite radius on the surface, it cannot represent a compact star.

Let us now consider the second factor of Eq. (21) which in its vanishing form ultimately provides the Schwarzschild [3] interior solution

$$ds^2 = -\left(1 - \frac{r^2}{R^2}\right)^{-1} - r^2(d\theta^2 + sin^2\theta d\phi^2) + \left(A + B\sqrt{1 - \frac{r^2}{R^2}}\right)^2 dt^2, \tag{25}$$

with its pressure and density as follows:

$$\kappa p = -\frac{A + 3B\sqrt{1 - \frac{r^2}{R^2}}}{R^2\left(A + B\sqrt{1 - \frac{r^2}{R^2}}\right)}, \tag{26}$$

$$\kappa \rho = \frac{3}{R^2}, \tag{27}$$

where A and R are non-zero constant quantities and $B > 0$.

If the mass function for electrically charged fluid sphere is denoted by $m(r)$, then it can be defined in terms of the metric function $e^{\lambda(r)}$ as

$$e^{-\lambda(r)} = 1 - \frac{2m(r)}{r} + \frac{q^2}{r^2}, \tag{28}$$

where the function $m(r)$ represents the gravitational mass of the matter contained in a sphere of radius r. Now, if R represents the radius of the fluid sphere then it can be shown that m is a constant with $m(r = R) = M$ outside the fluid distribution where M is the gravitational mass. Following the work of Florides [30] this can be defined as

$$M = \mu(R) + \xi(R), \tag{29}$$

where $\mu(R) = \frac{\kappa}{2}\int_0^R \rho r^2 dr$ is the mass inside the sphere, $\xi(R) = \frac{\kappa}{2}\int_0^R \sigma r q e^{\lambda/2} dr$ is the mass equivalence of the electromagnetic energy of distribution and $Q = q(R)$ is the total charge inside the fluid sphere.

By using Eq. (29) one can write the mass, in terms of energy density and charge function, as follows:

$$m(r) = \frac{\kappa}{2}\int \rho r^2 dr + \frac{1}{2}\int \frac{q^2}{r^2}dr + \frac{q^2}{2r}, \tag{30}$$

Again from Eqs. (15) and (18) we obtain expression for metric potential

$$\nu' = \frac{\left(\kappa r p + \frac{2m}{r^2} - \frac{2q^2}{r^3}\right)}{\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)}. \tag{31}$$

Also, the expression for the pressure, in its gradient form, can be obtained by using Eqs. (15) and (18) - (20) as follows

$$\frac{dp}{dr}=-\frac{M_G(r)\,(p+\rho)}{r^2}e^{(\lambda-\nu)/2}+\frac{q}{4\pi r^4}\frac{dq}{dr}, \tag{32}$$

where M_G is the gravitational mass within the sphere of radius r and is given by

$$M_G(r)=\frac{1}{2}r^2\nu' e^{(\nu-\lambda)/2}. \tag{33}$$

The above Eq. (32) represents the charged generalization of the Tolman-Oppenheimer-Volkoff (TOV) equation of continuity for perfect fluid stellar system [41, 42].

3 Algorithm for electromagnetic mass models

We are now in position to construct ‘electromagnetic mass’ models of stellar system for class one metric by using algorithm given by Maurya et al. [43].

The Eqs. (15) - (17) in terms of mass function reduce to

$$-\frac{2m(1+r\nu')}{r^3}+\frac{\nu'}{r}+\frac{q^2(1+r\nu')}{r^4}+\frac{q^2}{r^4}=\kappa p, \tag{34}$$

$$\begin{aligned}&-\frac{m'(2+r\nu')}{2r^2}-\frac{m(2r^2\nu''+r^2\nu'^2+r\nu'-2)}{2r^3}\\&+\frac{2rqq'\nu'-2q^2\nu'+4qq'+(r^2+q^2)(2r\nu''+r\nu'^2+2\nu)}{4r^3}-\frac{2q^2}{r^4}=\kappa p,\end{aligned} \tag{35}$$

$$\frac{2m'}{r^2}-\frac{2qq'}{r^3}=\kappa c^2\rho. \tag{36}$$

From Eqs. (34) and (35), the first order linear differential equation for $m(r)$ in terms of $\nu(r)$ and electric charge function $q(r)$ can be provided as follows:

$$m'+\frac{(2r^2\nu''+r^2\nu'^2-3\nu' r-6)}{r(r\nu'+2)}m=\frac{r(2r\nu''+r\nu'^2-2\nu')}{2(r\nu'+2)}+f(r), \tag{37}$$

where

$$f(r)=\frac{q^2\left[2r^2\nu''+r\nu'(r\nu'-4)-16\right]}{2r^2(r\nu'+2)}+\frac{qq'(r\nu'+2)}{r(r\nu'+2)}. \tag{38}$$

Hence the mass function $m(r)$ can be given by

$$m(r)=e^{-\int g(r)dr}\left[\int\{h(r)+f(r)\}\left(e^{\int g(r)dr}\right)dr+A\right], \tag{39}$$

where

$$g(r)=\frac{(2r^2\nu''+r^2\nu'^2-3r\nu'-6)}{r(r\nu'+2)}, \tag{40}$$

and

$$h(r)=\frac{r(2r^2\nu''+r\nu'^2-2\nu'}{2(r\nu'+2)}. \tag{41}$$

4 New class of electromagnetic mass models for stellar systems

4.1 Solution of Type I

We consider the following suitable function

$$\nu(r) = 2Ar^2 + logB, \tag{42}$$

$$\lambda(r) = log\left(1 + K\frac{\nu'^2 e^{\nu}}{4}\right), \tag{43}$$

where A and B are positive constants.

The expressions for the mass and the electric charge are respectively

$$\frac{2m(r)}{r} = Ar^2\left[\frac{De^{2Ar^2}}{1+DAr^2e^{2Ar^2}} + \frac{Ar^2(D^2e^{4Ar^2}+4-4De^{2Ar^2})}{2(1+DAr^2e^{2Ar^2})^2}\right], \tag{44}$$

$$\frac{q^2}{r^4} = E^2 = A^2r^2\left[\frac{D^2e^{2Ar^2}+4-4De^{2Ar^2}}{2(1+DAr^2e^{2Ar^2})^2}\right], \tag{45}$$

where $D = 4ABK$ is a pure constant.

Again, the expression for the energy density and the pressure are given by

$$8\pi\rho = A\left[\frac{D^2Ar^2e^{4Ar^2} - 4Ar^2 + 6De^{2Ar^2}(2Ar^2+1)}{2(1+DAr^2e^{2Ar^2})^2}\right], \tag{46}$$

$$8\pi p = A\left[\frac{-D^2Ar^2e^{4Ar^2} + 4(2+Ar^2) + 2De^{2Ar^2}(2Ar^2-1)}{2(1+DAr^2e^{2Ar^2})^2}\right]. \tag{47}$$

The respective gradients of above physical parameters are

$$\frac{dp}{dr} = -\frac{2A^2r}{8\pi}\left[\frac{-D^3Ar^2e^{6Ar^2} + D^2e^{4Ar^2}(-3+4Ar^2+8A^2r^4) + 4De^{2Ar^2}(4+7Ar^2+4A^2r^4) - 4}{2(1+DAr^2e^{2Ar^2})^3}\right], \tag{48}$$

$$\frac{d\rho}{dr} = -\frac{2A^2r}{8\pi}\left[\frac{D^3Ar^2e^{6Ar^2} + D^2e^{4Ar^2}(11+20Ar^2+28A^2r^4) - 4De^{2Ar^2}(6+7Ar^2+4A^2r^4) + 4}{2(1+DAr^2e^{2Ar^2})^3}\right]. \tag{49}$$

4.2 Solution of Type II

Here the functional relation for the metric potentials are

$$\nu(r) = 2log(1 + \sinh Ar^2) + logB, \tag{50}$$

$$e^{\lambda(r)} = \left(1 + K\frac{\nu'^2 e^{\nu}}{4}\right), \tag{51}$$

where A and B are positive constants.

The expressions of mass and electric charge are

$$\frac{2m(r)}{r} = Ar^2 \frac{D\cosh^2 Ar^2}{(1 + DAr^2\cosh^2 Ar^2)}$$
$$-Ar^2\left(\frac{D(2\sinh Ar^2\cosh Ar^2(1+\sinh Ar^2) + 2\cosh^3 Ar^2) - D^2\cosh^4 Ar^2(1+\sinh Ar^2) - 4\sinh Ar^2}{2(1+DAr^2\cosh^2 Ar^2)^2(1+\sinh Ar^2)}\right) \tag{52}$$

$$\frac{q^2}{r^4} = E^2 = A^2r^2\left(\frac{-D\left[(2\sinh Ar^2\cosh Ar^2 - D\cosh^4 Ar^2)(1+\sinh Ar^2) + 2\cosh^3 Ar^2\right] + 4\sinh Ar^2}{2(1+DAr^2\cosh^2 Ar^2)^2(1+\sinh Ar^2)}\right) \tag{53}$$

The energy density and pressure (taking $x = Cr^2$) are given by

$$8\pi\rho = A\frac{-4x\sinh x + D\left[2x\cosh^3 x + (6\cosh^2 x + 10x\sinh x\cosh x + Dx\cosh^4 x)(1+\sinh x)\right]}{2(1+Dx\cosh^2 x)^2(1+\sinh x)}, \tag{54}$$

$$8\pi p = A\frac{8\cosh x + 4x\sinh x + D\left[6x\cosh^3 x - (2\cosh^2 x + 2x\sinh x\cosh x + Dx\cosh^4 x)(1+\sinh x)\right]}{2(1+Dx\cosh^2 x)^2(1+\sinh x)}. \tag{55}$$

As done in the previous case, the expressions for the pressure and the density gradient can be determined by taking their derivatives with respect to r which are not produced here being their very complicated forms.

4.3 Solution of Type III

The metric potentials in this case are related to the following functions

$$\nu(r) = 2log(1 + \sin Ar^2) + logB, \tag{56}$$

$$e^{\lambda(r)} = \left(1 + K\frac{\nu'^2 e^{\nu}}{4},\right) \tag{57}$$

where A and B are positive constant.

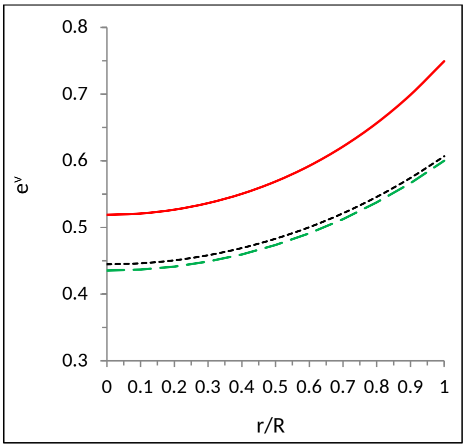

Fig. 1 e^{ν} are plotted with continuous line for solution I, small dashed line for solution II and long dashed line for solution III.

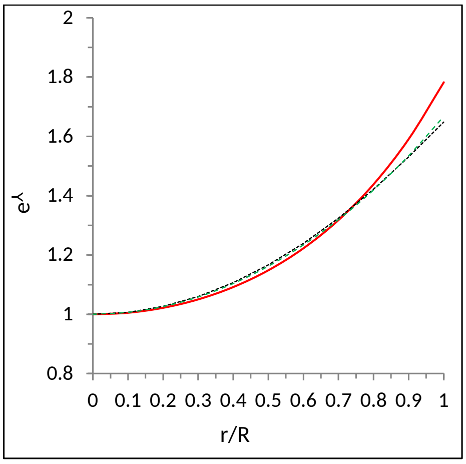

Fig. 2 e^{λ} are plotted with continuous line for solution I, small dashed line for solution II and long dashed line for solution III.

The expressions of mass and electric charge are

$$\frac{2m(r)}{r} = Ar^2 \frac{D\cos^2 Ar^2}{(1+DAr^2\cos^2 Ar^2)}$$
$$+Ar^2\left(\frac{D(\sin 2Ar^2(1+\sin Ar^2)-2\cos^3 Ar^2)+D^2\cos^4 Ar^2(1+\sin Ar^2)-4\sin Ar^2}{2(1+DAr^2\cos^2 Ar^2)^2(1+\sin Ar^2)}\right) \tag{58}$$

$$\frac{q^2}{r^4} = E^2 = A^2r^2\left(\frac{D\left[\sin 2Ar^2(1+\sin Ar^2)-2\cos^3 Ar^2\right]+D^2\cos^4 Ar^2(1+\sin Ar^2)-4\sin Ar^2}{2(1+DAr^2\cos^2 Ar^2)^2(1+\sin Ar^2)}\right) \tag{59}$$

The expression for energy density and pressure (taking $x = Cr^2$) are respectively

$$8\pi\rho = A\frac{4x\sin x + D\left[2x\cos^3 x + (6\cos^2 x - 5x\sin x)(1+\sin x)\right] + D^2 x\cos^4 x(1+\sin x)}{2(1+Dx\cos^2 x)^2(1+\sin x)}, \tag{60}$$

$$8\pi p = A\frac{8\cos x - 4x\sin x + D\left[6x\cos^3 x - (2\cos^2 x - x\sin 2x + Dx\cos^4 x)(1+\sin x)\right]}{2(1+Dx\cos^2 x)^2(1+\sin x)}, \tag{61}$$

Likewise the case II, the expressions for the pressure and the density gradients being very cumbersome we are leaving those calculations of respective derivatives.

Let us look at Figs. 1 and 2 regarding the desirable features on the basis of their respective solution. It is expected that the solution should be free from physical and geometrical singularities, i.e. the fluid pressure and the energy density at the center should be finite and metric potentials $e^{\lambda(r)}$ and $e^{\nu(r)}$ should have non-zero positive values in the range $0 \leq r \leq R$. At the center one must have $e^{\lambda(0)} = 1$ and $e^{\nu(0)} = B$ for each solution. Interestingly, both Figs. 1 and 2 show that metric potentials are positive and finite at the center.

Similarly, the density ρ should be positive and the pressure p must be positive inside the star as well as it should be zero at the boundary of the fluid sphere. All these features are quite available from Figs. 3 and 4.

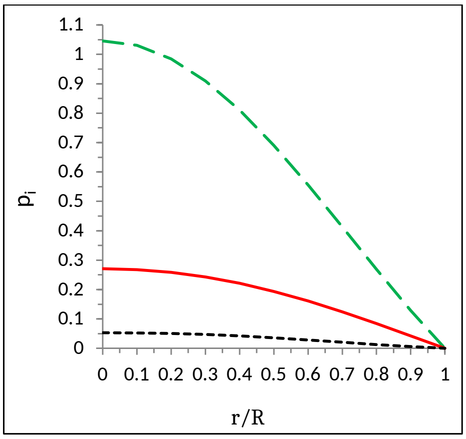

Fig. 3 Pressure is plotted with long dashed line for solution-I, continuous line for solution II and small dashed line for solution III.

Let us summarize the above results and at a glance try to get the flavor of these. We would like to mention here that if $A = 0$ in the cases (I), (II) and (III) then the corresponding metrics at once turn to flat spacetime and also the expressions for the electric charge, the pressure and the energy density automatically vanish. Therefore, the three charged fluid distributions obtained above depict the three electromagnetic mass models.

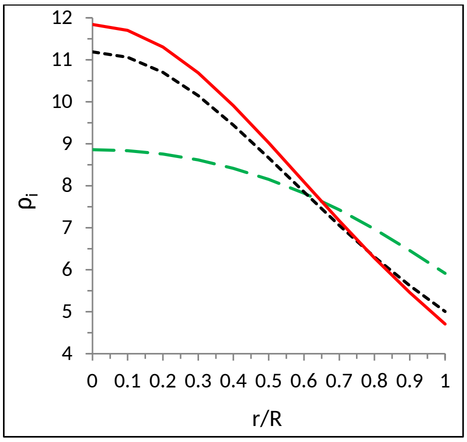

Fig. 4 Density is plotted with long dashed line for solution I, small dashed line for solution II, continuous line for solution III.

5 Boundary conditions for the spherical system

The above system of equations has to be solved under the condition that the radial pressure $p = 0$ at $r = a$ (where $r = a$ is the outer boundary of the fluid sphere). The interior metric (2) can join smoothly at the surface of spheres to the Reissner-Nordström metric [44]

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2. \quad (62)$$

This requires the continuity of $e^{\lambda}(r)$, $e^{\nu}(r)$ and $q(r)$ across the boundary $r = R$.

$$e^{-\lambda(R)} = \left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right), \quad (63)$$

$$e^{\nu(R)} = \left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right), \quad (64)$$

$$q(R) = Q, \quad (65)$$

$$p_{(r=R)} = 0. \quad (66)$$

By using all the above boundary conditions we are able to find out expressions for various constants as can be seen below.

5.1 Solution of Type I

$$D=\frac{(2AR^2-1)\pm\sqrt{(8A^2R^4+4AR^2+1)}}{AR^2e^{2AR^2}} \tag{67}$$

At the boundary:

$$e^{-\lambda(R)}=e^{\nu(R)}, \tag{68}$$

gives

$$B=\frac{1}{e^{2AR^2}(1+DAR^2e^{2AR^2})}. \tag{69}$$

5.2 Solution of Type II

The pressure is zero on the boundary $r=R$ (here $X=AR^2$) and we obtain expressions for the constants as follows:

$$D=\frac{D_1(X)\pm\frac{1}{2}\sqrt{16X\cosh^4X(1+\sinh X)(2\cosh X+X\sinh X)-D_2(X)}}{-X\cosh^4X(1+\sinh X)}, \tag{70}$$

where

$$D_1(X)=\cosh^2X(1-3X\cosh X+\sinh X)+X\cosh X\sinh X(1+\sinh X) \tag{71}$$

and

$$D_2(X)=-\cosh^2X\,(4X-2\cosh X+2X\cos 2X-2X\sin X-\sinh 2X)^2\,. \tag{72}$$

At the boundary:

$$e^{-\lambda(R)}=e^{\nu(R)}, \tag{73}$$

gives

$$B=\frac{1}{(1+\sinh X)^2\left(1+DX\cosh^2X\right)}. \tag{74}$$

5.3 Solution of Type III

The pressure being zero on the boundary $r=R$ (here $X=AR^2$) the constants can be given by

$$D=\frac{\cos^2X(6X\cos X-2\sin X-2)+X\sin 2X(1+\sin X)}{2X\cos^4X(1+\sin X)}, \tag{75}$$

$$\frac{\pm\sqrt{-16X\cos^4X(1+\sin X)(-2\cos X+X\sin X)+[6X\cos^3X-(2\cos^2X-X\sin 2X)(1+\sin X)]^2}}{-2X\cos^4X(1+\sin X)}. \tag{76}$$

At the boundary:

$$e^{-\lambda(R)}=e^{\nu(R)}, \tag{77}$$

gives

$$B=\frac{1}{(1+\sin X)^2\,(1+DX\cos^2X)}. \tag{78}$$

6 Physical features of the electromagnetic mass models for stellar systems

In the solution part (Sect. 4) we have analyzed some of the physical parameters, potentials, density, pressure etc., through their graphical plots. They exhibited desirable physical features regarding stellar configuration. However, in the present Sect. 6 we are interested to perform a few rigorous tests for other physical parameters, velocity and charge, and also prepare a check list for energy conditions and stability issues (such as TOV equation and Buchdahl condition).

6.1 Sound velocity

The velocity of sound should monotonically decrease away from the center and increase with the increase of density, i.e. $\frac{d}{dr}\left(\frac{dp}{d\rho}\right) < 0$ or $\frac{d^2p}{d\rho^2} > 0$ for $0 \leq r \leq R$. It is argued by Canuto [45] that the equation of state at ultra-high distribution of matter the sound speed decreases outwards.

In the present model, from Fig. 5, it is clear that velocity is decreasing for solution I and increasing for solution II and III throughout the star. Therefore, the solutions for solution II and III are not suitable at all as far as compact star is concerned. This is because the equation of state for nuclear matter shows a regular behavior of $\frac{dp}{d\rho}$ for these solutions [46].

The above discussions, based on the demonstration of the figures, immediately restraint ourselves to study henceforth only the solution of type I electromagnetic mass model.

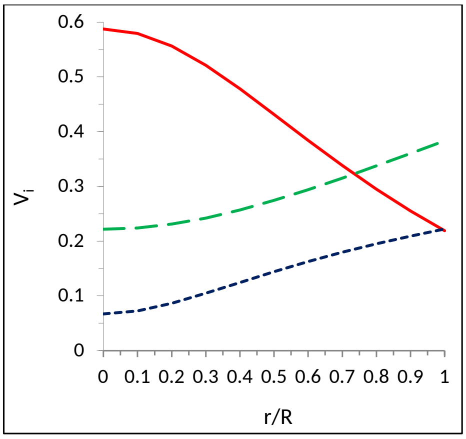

Fig. 5 Velocity is plotted with continuous line for solution I, long dashed line for solution II and small dashed line for solution III.

6.2 Electric charge for solution I

From the present model it is observed that in the unit of Coulomb, the charge on the boundary is 1.15295×10^{20} C and at the center it is zero (as the charge on

the boundary is 0.9889 so we have to multiply this by the number 1.1659×10^{20} to obtain the resultant numerical value).

One can observe from Fig. 6 that the charge profile starts from a minimum and acquires the maximum value at the boundary. This figure has been drawn for the compact star $RX\ J\ 1856-37$ with the constant values $CR^2 = 0.1836$, $D = 2.9540$.

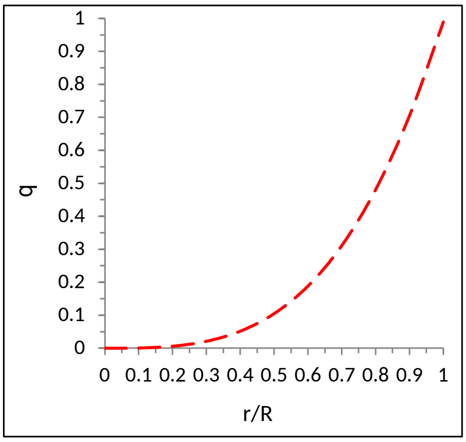

Fig. 6 Electric Charge is plotted with long dashed line for solution I.

6.3 Energy Conditions for solution of type I

For physical validity an energy-momentum tensor has to obey the following energy conditions:

1. null energy condition (NEC): $\rho + \frac{E^2}{4\pi} \geq 0$,
2. weak energy condition (WEC): $\rho - p + \frac{E^2}{4\pi} \geq 0$,
3. strong energy condition (SEC): $\rho - 3p + \frac{E^2}{4\pi} \geq 0$.

We have plotted the feature of different energy conditions in Fig. 7 for the values of different physical parameters connected to energy conditions for the constants: $CR^2 = 0.1836$, $M = 0.9041\ M_{\odot}$, $R = 6.006\ Km$ and $\frac{M}{R} = 0.222$. The figure indicates that all the energy conditions are satisfied throughout the interior region of the stellar system.

6.4 Generalized TOV equation for solution of type I

We write the generalized Tolman-Oppenheimer-Volkoff (TOV) equation [47] in the following form:

$$-\frac{M_G(\rho + p_r)}{r^2} e^{(\lambda-\nu)/2} - \frac{dp}{dr} + \sigma \frac{q}{r^2} e^{\lambda/2} = 0, \tag{79}$$

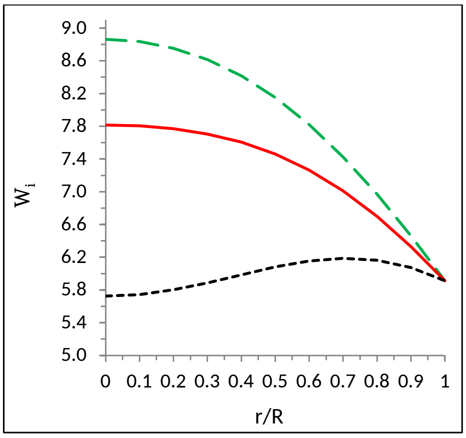

Fig. 7 NEC is plotted with long dashed line, WEC is plotted with continuous line and SEC is plotted with small dashed line for solution I.

where M_G is the effective gravitational mass within the radius r and can be provided

$$M_G(r) = \frac{1}{2} r^2 \nu' e^{(\nu-\lambda)/2}. \tag{80}$$

The above TOV equation describes the equilibrium condition for a charged fluid subject to gravitational (F_g), hydrostatic (F_h) and electric (F_e) forces. Therefore, one can write it in a more suitable form

$$F_g + F_h + F_e = 0, \tag{81}$$

where

$$F_g = -\frac{1}{2}\nu'(\rho + p) = -\frac{2A^2 r}{8\pi}\left[\frac{2De^{2Ar^2}(1+4Ar^2)+4}{(1+DAr^2e^{2Ar^2})^2}\right], \tag{82}$$

$$F_h = -\frac{dp}{dr} = \frac{2A^2 r}{8\pi}\left[\frac{-D^3Ar^2e^{6Ar^2} + D^2e^{4Ar^2}(-3+4Ar^2+8A^2r^4) + 4De^{2Ar^2}(4+7Ar^2+4A^2r^4) - 4}{2(1+DAr^2e^{2Ar^2})^3}\right], \tag{83}$$

and

$$F_e = \sigma\frac{q}{r^2}e^{\lambda/2} = \frac{A^2 r}{4\pi}\left[\frac{(-2+De^{Ar^2})\left[-6+D^2Ar^2e^{4Ar^2} + De^{2Ar^2}(3+2Ar^2+8A^2r^4)\right]}{2(1+DAr^2e^{2Ar^2})^3}\right], \tag{84}$$

The plot for the TOV equation is shown in Fig. 8. We observe from this figure that the system under the joint balancing action of the different forces, e.g. gravitational, hydrostatic and electric to attain an overall static equilibrium. However, from Fig. 8 it is also clear that the gravitational force has the dominant role over the hydrostatic force whereas the electric force has the negligible contribution to the equilibrium. This feature seems quite reasonable in the case of the compact stellar system.

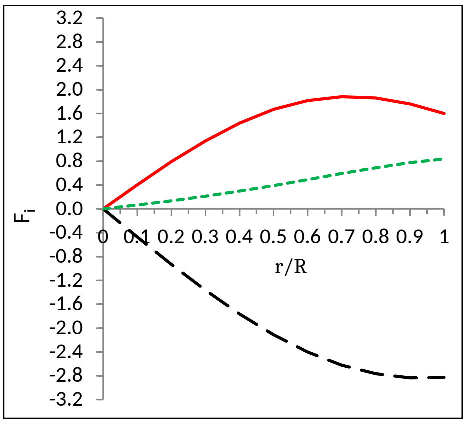

Fig. 8 F_g is plotted with long dashed line, F_h is plotted with continuous line and F_e is plotted with small dashed line for solution I.

6.5 Effective mass-radius relation and surface redshift for solution of type I

Buchdahl [48] has proposed an absolute constraint of the maximally allowable mass-to-radius ratio (M/R) for isotropic fluid spheres in the form $2M/R \leq 8/9$. However, Böhmer and Harko [49] have shown that for a compact object with charge, $Q(< M)$, there is a lower bound for the mass-radius ratio

$$\frac{3Q^2}{2R^2}\left(\frac{1+\frac{Q^2}{18R^2}}{1+\frac{Q^2}{12R^2}}\right) \leq \frac{2M}{R}, \tag{85}$$

whereas the upper bound of the mass-radius of a charged sphere was generalized by Andreasson [50] as follows:

$$\sqrt{M} \leq \frac{\sqrt{R}}{3} + \sqrt{\frac{R}{9} + \frac{Q^2}{3R}}. \tag{86}$$

In the present model, the effective gravitational mass is given by

$$M_{eff} = 4\pi \int_0^R \left(\rho + \frac{E^2}{8\pi}\right) r^2 dr = \frac{1}{2}R[1 - e^{-\lambda(R)}] = \frac{1}{2}R\left[\frac{DAR^2e^{2AR^2}}{1+DAR^2e^{2AR^2}}\right]. \tag{87}$$

Therefore, the compactness factor can be written as

$$u = \frac{M_{eff}}{R} = \frac{1}{2}\left[\frac{DAR^2e^{2AR^2}}{1+DAR^2e^{2AR^2}}\right]. \tag{88}$$

The surface redshift in connection with the above compactness is given by

$$Z = (1-2u)^{-1/2} - 1 = e^{\lambda(R)/2} - 1 = \sqrt{1+DAR^2e^{2AR^2}} - 1. \tag{89}$$

The plot of the surface redshift is shown in Fig. 9 for the compact star $RX\ J\ 1856-37$ with the constant values $CR^2 = 0.1836$, $D = 2.9540$. It can be observed that

there is a gradual increase in the redshift which is an acceptable physical feature. The maximum surface redshift for the present stellar configuration of radius $R = 6.006$ Km turns out to be $Z = 0.3882$ which seems well within the limit $Z \leq 2$ [48,51,52].

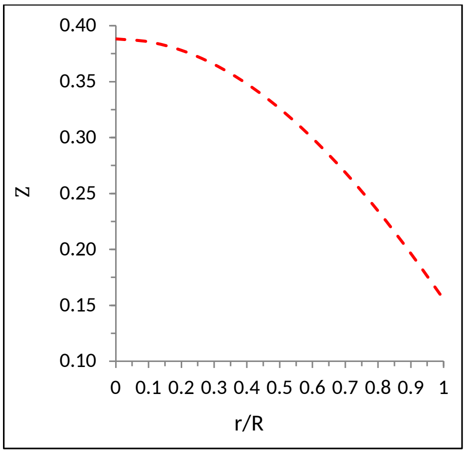

Fig. 9 Red shift is plotted with long dashed line for solution I.

7 Validating the model with strange star candidates

In the foregoing Sect. 6 we have studied several physical behavior of stellar system in connection with electromagnetic mass models. In some of the subsections, e.g. 6.2 (electric charge) and 6.5 (surface redshift), we have also shown graphical plots specifically for the compact star $RX\ J\ 1856-37$ with the mass $M = 0.9041\ M_{\odot}$ and the radius $R = 6.006$ Km.

However, it seems that some more investigations are needed to show the validity of our models for other compact stars which have definite observed physical features. In the following two Tables 1 and 2 we, therefore, produce data sheet for the purpose of comparison between the present model stars and the observed compact stars.

Table 1 Values of the model parameters A, B, D and K for different strange stars

Strange star candidates	M ($M_{\odot}$)	R (Km)	M/R	B	A	D	K
$RX\ J\ 1856-37$	0.9041	6.006	0.222	0.5189	5.0962×10^{-13}	2.9540	2.7925×10^{12}
$Her\ X-1$	0.9825	6.700	0.216	0.5552	3.6319×10^{-13}	3.0626	3.7972×10^{12}
$PSR\ 1937+21$	2.1	11.4998	0.269	0.4103	1.9503×10^{-13}	2.5857	8.0775×10^{12}
$PSRJ\ 1614-2230$	1.97	11.3664	0.2553	0.4419	1.8112×10^{-13}	2.6998	8.4338×10^{12}
$PSRJ\ 0348+0432$	2.1	11.7372	0.2636	0.4228	1.8017×10^{-13}	2.6315	8.6369×10^{12}

Table 2 Energy densities and pressure for different strange star candidates for the above parameter values of Table 5

Strange star candidates	Central Density (gm/cm^{-3})	Surface density (gm/cm^{-3})	Central pressure $(dyne/cm^{-2})$
RXJ 1856 − 37	2.4252×10^{15}	1.6183×10^{15}	2.2243×10^{35}
$Her\ X-1$	1.8869×10^{15}	1.2718×10^{15}	2.5768×10^{35}
PSR 1937 + 21	8.1241×10^{14}	5.0473×10^{14}	1.3334×10^{35}
$PSRJ$ 1614 − 2230	7.4524×10^{14}	4.9876×10^{14}	1.1384×10^{35}
$PSRJ$ 0348 + 0432	7.6379×10^{14}	4.7797×10^{14}	1.1918×10^{35}

For our model we particularly note that for the compact star $RXJ\ 1856 - 37$ with mass $M = 0.9041\ M_{\odot}$ and radius $R = 6.006$ Km the surface redshift turns out to be $Z = 0.3882$ which seems falls within the range $Z \leq 2$ [48,51,52] and $0 < Z \leq 1$ [53,54,55,56,57]. However, one may figure out the surface redshifts for other compact stars also as provided in Tables 1 and 2 and we expect those values will be within the above specified range. On the other hand, surface density as can be seen from Table 2 is of the order of $10^{14} - 10^{15}$ gm/cc. This very high density indicates that the model under 'electromagnetic mass' represents an ultra-compact star [58,59,60].

Therefore, we would like to pass a general remark that our models in connection with 'electromagnetic mass' represent compact stars of several categories.

8 Conclusion

We have considered the static spherically symmetric spacetime metric of embedding class one in the present investigation. Is has been possible to show the existence of electromagnetic mass models specifically in connection with compact stars. Three new electromagnetic mass models are derived where the solutions are entirely originating from the electromagnetic field, such that the density and pressure like physical parameters do vanish for the vanishing charge alone. However, a meticulous analysis reveals that among these three sets of solutions all are not equally interesting as far as astrophysical several aspects are concerned. To validate these special type of solutions related to electromagnetic mass models, we have also conducted a comparison between our proposed model and the observed compact stars which shows satisfactory results in favor of the present theoretical modeling.

However, an obvious question may arise to the study of the compact stellar configuration under Einstein-Maxwell spacetime, especially how the charge comes in the consideration of such kind of systems. A brief historical note on the issues of stability of static spherically symmetric stellar systems and as an effective measure of averting singularity why one should include charge and what is the process of holding huge amount of charge inside the bodies are available exhaustively in the Ref. [35] and the Refs. therein. As an continuation of this discussion, we feel, an outline on the charged bodies may be helpful to the readers which follows in the next paragraph.

In the history of general relativity the first ever exact solutions of the Einstein field equations, the well-known Schwarzschild interior solutions, suffer from the

problem of singularity due to gravitational collapsing of a spherically symmetric matter distribution. One way to overcome this singularity is to include electrical charge to the neutral bodies. It is suggested by the scientists that gravitational collapse can be avoided in the presence of charge where the gravitational attraction is counter balanced by the electrical repulsion in addition to the pressure gradient [61, 62, 63]. To this end questions came up regarding the stability of the charged sphere and also about the amount of charge that holds by the star. A good amount of works have been done by several authors on the stability issue [64, 65, 66, 67, 68, 69]. On the other hand, in some recent studies [70, 71, 47] we find out estimate of electric charge in the compact stars which amounts a huge charge of the order of $10^{19} - 10^{20}$ Coulomb.

Acknowledgments

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